- **Degrees of freedom of some Random Fourier Matrices.**

- Aline Bonami - Université d'Orléans

This work originates in the study of channel matrices of particular wireless networks of telecommunication, which are $n \times n$ matrices with coefficients

$$a_{jk} = \frac{e^{2i\pi|r_j - r'_k|/\lambda}}{|r_j - r'_k|}.$$

Here r_j is the position of the $j-$th emitter node while r'_k is the position of the $k-$th receptor node. λ denoted the wavelength. Both $n-$tuples of nodes are chosen randomly in two squares situated at large distance. This model has been considered by Desgroseilliers, Lévêque and Preissmann, who are in particular interested by the number of degrees of freedom of this system. Moreover they replace this matrix by a random Fourier matrix, which can be justified under particular conditions on the distance between squares and their side. More generally, we will consider $n \times n$ matrices A with entries

$$a_{j,k} = \frac{1}{n} \exp(2i\pi X_j Y_k),$$

where X_j, Y_k, $j, k = 1, \ldots, n$ are $2n$ independent random variables. We assume that the X_j's follow the same probability law P, while the Y_k's follow the law Q. Concentration inequalities allow us to compare the spectrum of A^*A with the one of the integral operator on $L^2(Q)$ whose kernel κ is given in terms of the characteristic function of the law P. Comparison is given in terms of ℓ^2 norms. When P and Q are uniform laws, respectively on $(-m/2, m/2)$ and $(-1/2, +1/2)$, we recover the sinc kernel with parameter m,

$$\kappa(x, y) = \frac{\sin(\pi m(x - y))}{\pi m(x - y)}.$$

In this case, up to a change of variables, the matrix A may be seen as a discretisation of the finite Fourier transform, defined on $L^2(-1/2, +1/2)$ by

$$\mathcal{F}_m(f)(x) := \int_{-1/2}^{+1/2} e^{2i\pi mxy} f(y) dy.$$

The number of degrees of freedom of the matrix A at level ε and confidence level α is defined as

$$\deg_\infty(A, \varepsilon, \alpha) = \min\{s; \lambda_s(A) \leq \varepsilon \quad \text{with probability } \geq \alpha\}.$$

Here $\lambda_s(A)$ denotes the non increasing sequence of the singular values of A. We prove that m is a good approximation of $\deg_\infty(A, \varepsilon, \alpha)$ in our particular case for m and n/m large. Generalizations to other probability laws will also be given.

This is a joint work with Abderrazek Karoui.

- **Convergence Rates of Random Walk Approximations of Backward SDEs - Christel Geiss -** University of Jyväskylä

  For the forward backward SDE

  $$X_t = x + \int_0^t b(r, X_r)dr + \int_0^t \sigma(r, X_r)dB_r$$

  $$Y_t = g(X_T) + \int_t^T f(s, X_s, Y_s, Z_s)ds - \int_t^T Z_s dB_s, \quad 0 \le t \le T$$

  Briand, Delyon and Memin have shown in 2001 a Donsker-type theorem: If one approximates the Brownian motion B by a random walk B^n, the according solutions (X^n, Y^n, Z^n) converge weakly to (X, Y, Z).

  If the random walk is constructed from the underlying Brownian motion by Skorohod embedding it can be shown that (X_t^n, Y_t^n, Z_t^n) converges to (X_t, Y_t, Z_t) in L_2. We estimate the rate of convergence in dependence of smoothness properties of the coefficients b and σ, the terminal condition function g and the generator f. For this we need growth and smoothness properties of the PDE associated to the FBSDE as well as of the difference equations associated to the approximating stochastic equations. We derive these properties by stochastic methods.

  The talk is based on joint results with Céline Labart and Antti Luoto

- **Random sections of ellipsoids and the power of random information - Aicke Hinrichs -** Johannes Kepler Universität

  We study the circumradius of the intersection of an m-dimensional ellipsoid $\mathcal{E}$ with semi-axes $\sigma_1 \ge \cdots \ge \sigma_m$ with random subspaces of codimension n. We find that, under certain assumptions on σ, this random radius $\mathcal{R}_n = \mathcal{R}_n(\sigma)$ is of the same order as the minimal such radius σ_{n+1} with high probability. In other situations $\mathcal{R}_n$ is close to the maximum σ_1. The random variable $\mathcal{R}_n$ naturally corresponds to the worst-case error of the best algorithm based on random information for L_2-approximation of functions from a compactly embedded Hilbert space H with unit ball $\mathcal{E}$.

  In particular, σ_k is the kth largest singular value of the embedding $H \hookrightarrow L_2$. In this formulation, one can also consider the case $m = \infty$, and we prove that random information behaves very differently depending on whether $\sigma \in \ell_2$ or not. For $\sigma \notin \ell_2$ random information is completely useless. For $\sigma \in \ell_2$ the expected radius of random information tends to zero at least at rate $o(1/\sqrt{n})$ as $n \to \infty$.

  In the proofs we use a comparison result for Gaussian processes à la Gordon, exponential estimates for sums of chi-squared random variables, and estimates for the extreme singular values of (structured) Gaussian random matrices.

  This is joint work with David Krieg, Erich Novak, Joscha Prochno and Mario Ullrich

- **Average case approximation complexity as $d \to \infty$. Old and new results**

  - Alexey Khartov - St.Petersburg State University

  We study approximation properties of centered second order random fields $Y_d(t)$, where $d \in \mathbb{N}$ is a dimension of the parameter t. The *average case approximation complexity* $n^{Y_d}(\varepsilon)$ is defined as the minimal number of evaluations of arbitrary linear functionals that is needed to approximate Y_d with normalized 2-average error not exceeding a given threshold $\varepsilon \in (0,1)$. We consider the quantity $n^{Y_d}(\varepsilon)$ for fixed ε and $d \to \infty$ and investigate its asymptotic behaviour. In the talk we will review old and new general results for this setting and propose examples of their applications.

  We will focus on the results obtained in joint works with Marguerite Zani.

- **How good are random sampling points?**

  - David Krieg - Johannes Kepler Universität

  The solution $S(f)$ of a numerical problem often depends on an unknown function f. We want to compute $S(f)$ but our knowledge of f is restricted to basic properties of f and n function values.

  Usually, we assume that we can compute these function values at arbitrary points. We try to choose the points in a way that allows us to minimize the error. That is, we ask for optimal sampling points.

  In this talk, we assume that we do not get to choose the points. Instead, we imagine that the points are random and ask: How much do we loose in comaprison to optimal sampling points? We study this question for several examples where $S(f)$ is the integral of f or the function itself.

  This is joint work with Aicke Hinrichs, Erich Novak, Joscha Prochno and Mario Ullrich

- **Coding of Poisson random sets**

  - Mikhail Lifshits - St.Petersburg State University

  Consider a random set (or "picture") in the unit cube of d-dimensional Euclidean space as a union of balls centered at points of a Poissonian random field and having i.i.d. radii. Let K be the minimal number of balls needed to reproduce the picture.

  We study large deviation probabilities for K and prove in some cases that for large n we have $\ln P(K > n) \sim \ -An \ln n$ where the constant A may explicitly depend on dimension, on the distribution of radii, and on the norm under consideration. In many cases the problem of finding the value of A remains open although some upper and lower bounds are available.

  This asymptotics has natural corollaries in high dimensional quantization problems.

  This is a joint work with F. Aurzada (TU Darmstadt).

- **Relating direct and inverse Bayesian problems via the modulus of continuity**

  - Peter Mathe- Weierstrass Institute

  We shall analyze a Bayesian approach to inverse problems $y = F(x)$, for a class of non-linear mappings F in Hilbert space. In a recent study [2], the authors have indicated an intrinsic relation between the direct problem, where the goal is to find y based on noisy data y^σ, and the given inverse one, where the goal is to find x based on noisy data y^σ. This relation is the *modulus of continuity*, as this is well known, see e.g. [1], from the classical theory of inverse problems.

  In this talk we shall highlight how to unveil the form of the modulus of continuity, based on smoothness assumptions of the unknown solution x. Then we shall show how to solve the direct problem by using projection schemes. This then yields a corresponding solution to the inverse problem. We shall discuss optimality of the proposed approach.

  This is joint work with S. Agapiou, University of Cyprus, Nicosia

  [1] B. Hofmann, P. Mathé, and M. Schieck, Modulus of continuity for conditionally stable ill-posed problems in Hilbert space, J. Inverse Ill-Posed Probl. 16 (2008), no. 6, 567?585. MR2458286

  [2] Bartek Knapik and Jean-Bernard Salomond, A general approach to posterior contraction in nonparametric inverse problems, Bernoulli 24 (2018), no. 3, 2091?2121. MR 3757524

- **IBC for multivalued problems**

  - Leszek Plaskota - University of Warsaw

  Information-based compexity (IBC) deals with complexity of approximating problems (mappings) $S : F \to G$ based on incomplete information about problem instances $f \in F$. In this study, usually a single-valued mappings S are considered, despite the fact that sometimes there is a need to approximate multivalued mappings $\mathbf{S} : F \to 2^G$, i.e., where $\mathbf{S}(f)$ is a subset of G. A natural example is provided by the problem of finding all the zeros of a function. In this talk, we show how to extend the IBC theory to cover approximation of multivalued mappings, where the error between the exact solution $\mathbf{S}(f)$ and its (multivalued) approximation $\mathbf{A}(f)$ is measured by the *Hausdorff distance*. We consider different settings including worst case, average case, and asymptotic settings. We also provide sample results for zero finding and for function approximation and integration of 1-Lipschitz functions, based on their absolute values at finitely many points.

- **Sanov-type large deviations in the Schatten classes**

  -Joscha Prochno - Karl-Franzens-Universität

  Denote by $l_1(A), ..., l_n(A)$ the eigenvalues of an (nxn)-matrix A. Let Z_n be an (nxn)-matrix chosen uniformly at random from the matrix analogue to the classical l_p^n-ball, defined as the set of all self-adjoint (n x n)-matrices satisfying for which the l_p-norm of the vector of eigenvalues is less than or equal to 1. We present a large deviations principle for the random spectral measure of the matrix $n^(1/p)Z_n$. As a consequence, we obtain that the spectral measure of $n^(1/p)Z_n$ converges weakly almost surely to a non-random limiting measure given by the Ullman distribution. Corresponding results for random matrices in Schatten classes, where eigenvalues are replaced by the singular values, also hold.

  Joint work with Zakhar Kabluchko and Christoph Thäle

- **Construction of low-dispersion point sets from error-correcting codes**

  - Mario Ullrich - Johannes Kepler Universität

  In the last years there was an increasing interest in point sets with 'small' dispersion, i.e., in point sets that intersect each axis-parallel box with 'large' volume, especially in high dimensions. Despite the many existence results, there was also some progress regarding the actual construction. Here, we show how to generate point sets, whose size is optimal with respect to the dimension d, using certain error-correcting codes. The running-time of the underlying procedure depends polynomially on d.

- **Exponential Tractability for Weighted Tensor Product Problems**

  - Henryk Woźniakowski - Columbia University and University of Warsaw

  Tractability studies complexity of d-variate problems solved to within ε. So far, algebraic tractability was studied where we relate complexity to d and ε^{-1}. For C^∞ or analytic problems, exponential tractability has been recently studied where we relate complexity to d and $\ln \varepsilon^{-1}$. In this talk we show necessary and sufficient conditions on weights of tensor product problems to obtain various kinds of exponential tractability and compare them to conditions for algebraic tractability.

  Joint work with Peter Kritzer and Friedrich Pillichshammer